On the Vertical Structure of a Competitive Electricity Market

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Abstract: Electricity is a completely non-storable good that often faces uncertain demand. This paper studies how the vertical structure of a competitive electricity market affects its performance. In the model all firms are perfectly competitive and risk-neutral. The aggregate demand is stochastic but retail prices must be determined before the uncertainty is resolved. When producers and retailers are vertically separated, they trade with each other in a spot wholesale market. It is found that when the covariance between production quantities and marginal production costs is positive, vertical separation leads to higher prices for consumers and higher profits for producers in the short-run. The result is reversed when the covariance is negative. In the long-run, vertical separation may induce excessive or insufficient entry of producers, depending on the sign of the covariance. But in either case the separation leads to higher retail prices and lower social surplus.

Keywords: Demand uncertainty; Electricity; Non-storable goods; Vertical Integration

JEL Classification: L22, L51, L94

Introduction

Because of the natural monopoly characteristics of electricity industry, traditional electricity firms are normally subject to intensive government regulation. The firms are usually vertically integrated, engaging in generation, transmission, distribution, and retail of electricity. Government regulation precludes market competition, therefore often results in cost inefficiency. In the last 30 years, the deregulation of electricity industries has been focusing on designing appropriate market environment so that producers and retailers can effectively compete with each other, while keeping transmission and distribution businesses under regulation. The reforms often create vertical separated electricity industries where producers and retailers trade with each other in spot wholesale markets. Since the demand for electricity is stochastic, such an industry structure often results in highly fluctuating wholesale prices. Another possible deregulation approach is creating multiple competing electricity firms that directly serve final consumers. Such an industry structure eliminates spot wholesale markets. An interesting question is how the vertical structure of the industry influences the market performances.

This paper considers a perfectly competitive electricity market where the aggregate demand is stochastic. Electricity is completely non-storable. Electricity production incurs variable marginal costs, but retailing incurs constant marginal costs and zero fixed costs. Linear retail prices are determined before the demand uncertainty is resolved. A seller has to fully satisfy the demands from its contracted consumers. Under vertical integration, producers directly sell to final consumers. Under vertical separation, retailers purchase from producers through a spot wholesale market, and then sell to final consumers. The generation facilities of the industry are fixed in the short-run but are adjustable in the long-run.

This model suggests that when the covariance between electricity outputs and marginal production costs is positive, the separation of production and retail results in higher price for consumers and higher profits for producers in the short-run. Under vertical separation, the spot wholesale market serves as a commitment device for the producers to attain more revenue through a multiplicative effect. Indeed, the producer profits rise significantly when the aggregate demand is high because they not only sell more but also sell at higher wholesale prices. When the demand is low and thus the wholesale price is low, the loss is relatively small because the output is also low. The net effect of the uncertainty on the (expected) producer profits is positive compared to that under vertical integration. The result is reversed when the covariance is negative.

In the long-run producers enter or exit the market until their economic profits are driven to zero. Vertical separation induces excessive entry of producers to the industry when the covariance between production quantities and marginal costs is positive, because the separation exaggerates the profitability of the production business. The result is reversed when the covariance is negative. But in either case, the suboptimality of market entry leads to higher retail price, lower consumer surplus and lower social surplus. Therefore from the perspective of welfare economics, vertical separation might be inferior to vertical integration in the long-run.

There is a theoretical literature on the competition of electricity markets. Klemperer and Meyer (1989) and von der Fehr and Harbord (1993) consider oligopolistic markets where firms face uncertain demand. The firms compete by offering continuous (Klemperer et al., 1989) or discrete (von der Fehr et al., 1993) supply functions. Klemperer et al. (1989) find that in the absence of uncertainty, there exists an enormous multiplicity of equilibria in supply functions, but uncertainty dramatically reduces the set of equilibria. Under uncertainty, they prove the existence of a Nash equilibrium in supply functions for a symmetric oligopoly and give sufficient conditions for uniqueness. von der Fehr et al. (1993) study the price competition in the deregulated UK electricity market with an auction model. They show that under the existing institutional set-up there is likely to be above marginal cost pricing, and inefficient dispatching may result. It is also found that for a wide range of demand distributions, pure strategy equilibria will not exist with step supply functions. In contrast, the strategic interaction among firms does not exist in the current paper since the firms are price-takers.

Another important theoretic paper is Allaz and Vila (1993), which shows that the existence of future markets may increase the market efficiency in a Cournot setting. Through long-term contracts, generators sell part of their electricity before the opening of spot markets at fixed prices. As a result, generators will behave more competitively in the spot market. The spot market outcome approaches the perfectly competitive market outcome as the number of rounds of contracting gets large. Based on this theory, Bushnell, Mansur and Saravia (2008) simulate electricity prices that define bounds on static oligopoly equilibria, and find that vertically integrated wholesalers, or those with long-term contracts, have substantially less incentive to raise wholesale prices.

Gans and Wolak (2008) offer a prospective and retrospective quantitative assessment of the impact of a “passive” vertical integration between a large electricity retailer and a large electricity generator in the Australian National Electricity Market. The integration is “passive” in the sense that the acquiring retailer is required not to involve in the day-to-day bidding and contract trading of the acquired generator, with representation only at the Board of Directors level. The argument is that a passive vertical acquisition provides the acquiring retailer with a non-contractual natural hedge against fluctuations in the wholesale market. This causes the retailer to reduce its demand for fixed-price forward contracts which in turn reduces the total volume of such contracts held by generators. Therefore the generators increase their incentive to exercise unilateral market power in the spot wholesale electricity market, which raises the equilibrium market prices. A feature of these papers is that the industry has a “hybrid” vertical structure, where a producer can choose to sell through a spot wholesale market, or directly to final consumers.

Industrial organization theorists have paid considerable attention to vertical separation under aggregate demand uncertainty, *e.g.*, Deneckere, Marvel and Peck (1996), Dana and Spier (2001), and Marvel and Wang (2009). The studies show how vertical integration corrects the distortion caused by demand uncertainty. The models usually assume that the products in consideration can be produced in advance and inventoried by retailers for possible future sale. Hence the products can be stored for at least “one period”. In contrast, electricity cannot be inventoried at all, which means aggregate supply must match aggregate demand in real time. Nevertheless, the current paper also suggests that vertical integration improves the coordination between upstream and downstream firms when the aggregate demand is uncertain.

Led by Chile in 1982 and the United Kingdom in 1990, many countries have reformed their electricity industries in order to improve performances. Some of the reforms are viewed rather successful, *e.g.*, the Nordic market (including Denmark, Finland, Norway and Sweden). Amundsen and Bergman (2006) suggest that the successful reform in the Nordic market is attributed to a simple but sound market design, successful dilution of market power, strong political support of deregulation, and high proportion of hydroelectric energy.[1] Other reforms might be disappointing, *e.g.*, the British market (Green and Newbery 1992) and Californian market (Borenstein, Bushnell and Wolak, 2002; Joskow and Kahn, 2002) before 2001. Green and Newbery (1992) find that the competition in supply schedules in the British electricity spot market implies a high markup on marginal cost and substantial deadweight losses. Wolak (2003) "diagnoses" the Californian electricity crisis during 2000-2001 and finds that the Federal Energy Regulatory Commission (FERC) should regulate, rather than simply monitor, wholesale electricity markets.

The rest of the paper proceeds as follows. Section I presents a simple model of competitive electricity market. Section II characterizes the short-run equilibria of the market, and Section III concerns the long-run equilibria. Section IV discusses some possible extensions of the model. Section V concludes the paper.

I. An electricity market

Consider an electricity market that is perfectly competitive at both production and retail stages. The production cost of an electricity producer is $C(q)$, which is positive and

[1] Note that the assessment is not based on a quantitative assessment of the market outcomes before versus after the reform.

increasing. Denote $MC(q) \equiv C'(q)$ and $AC(q) \equiv \frac{C(q)}{q}$ as usual.[2] The marginal transmission and distribution costs through a public grid are normalized to zero. Retailers have zero fixed costs and constant marginal costs, which are also normalized to zero.[3] Electricity is assumed to be completely non-storable.

There is a continuum of consumers. Their demands for electricity are homogenous, stochastic, and completely inelastic. The individual demands are perfectly correlated with each other. The aggregate demand follows cumulative distribution $F(\cdot)$ on interval $[\underline{x}, \overline{x}] \subseteq R^{+}$. The expected aggregate demand is denoted by $N \equiv \int_{\underline{x}}^{\overline{x}} q dF(q)$. Without loss of generality, the number of consumers is assumed to be *N*. Hence the demand of each consumer follows distribution $F(Nx)$ on interval $[\underline{x}', \overline{x}'] \equiv [\frac{\underline{x}}{N}, \frac{\overline{x}}{N}]$, and her expected demand is exactly 1.

All firms are risk-neutral. The sale contracts are signed before the demand uncertainty is resolved. Consumers only accept fixed prices. A contract specifies a linear retail price, but not the quantity of transaction. A firm can choose the number of consumers to serve. Once the contracts are signed, the firm must fully satisfy the demand of its consumers. In the case of vertical integration, producers directly sell to final consumers. In the case of vertical separation, retailers purchase from producers through a spot wholesale market and then sell to consumers. In this model the role of competitive retailers is transforming the uncertain spot wholesale prices into a constant retail price.

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2 If the producer has a capacity limit *Q*, then we have $C'(q) = +\infty$ for $q > Q$.

3 This assumption is justified by the fact that electricity retailers mostly offer marketing and billing services that do not have significant increasing return to scales.

II. Short-run equilibrium

In the short-run, the number of firms and the production facility of the industry are fixed. We normalize the number of price-taking producers to 1 in order to simplify the explanation. This simplification is not restrictive as long as the producers are homogenous. I will briefly demonstrate how the model can be extended to the case with multiple producers at the end of this section.

2.1 Vertical integration

Because the consumer's demand is uncertain, an integrated electricity producer cannot determine the exact production quantity when the sale contracts are signed, but it can choose the number of consumers to serve. The equilibrium of the market can be characterized by a price p^I that clears the market. Since the consumer's demands are absolutely inelastic, the equilibrium price should be the lowest price that induces the producer to serve all the consumers.

In equilibrium the producer's expected revenue from serving the "last" consumer (*i.e.*, the expected marginal revenue) should be equal to the expected cost of serving that consumer (*i.e.*, the expected marginal cost). Since each consumer's expected demand is 1, the producer's expected marginal revenue is just p^I, and the expected marginal cost of serving the last consumer is $\int_{\underline{x}}^{\bar{x}} MC(q)dF(q)$. Hence the short-run equilibrium price p^I is

$$p^I = \int_{\underline{x}}^{\bar{x}} MC(q)dF(q). \tag{1}$$

The producer would be willing to serve more than $N \equiv \int_{\underline{x}}^{\bar{x}} qdF(q)$ consumers if the market price is higher than p^I, and *vice versa*. Noted that p^I must be given before the demand uncertainty is resolved. Thus it is a predetermined but not an "expected" selling price.

Given the equilibrium price, the producer's expected profit is

$$E(\pi^I) = \int_{\underline{x}}^{\overline{x}} qdF(q) \cdot \int_{\underline{x}}^{\overline{x}} MC(q)dF(q) - \int_{\underline{x}}^{\overline{x}} C(q)dF(q). \qquad (2)$$

2.2 Vertical separation

Under vertical separation there is a spot wholesale market where electricity producers and retailers trade with each other. The competitive spot wholesale price equals to the marginal production cost $MC(q)$, which is random since the quantity-demanded q is uncertain. If the aggregate demand turns out to be q, then the transaction value in the wholesale market is $qMC(q)$. Hence the producer's expected revenue, or the retailers' expected total cost, is $\int_{\underline{x}}^{\overline{x}} qMC(q)dF(q)$. Since the competitive retailers have constant return to scale, even the short-run equilibrium retail price p^S should allow the retailers to break even, *i.e.*,

$$p^S \int_{\underline{x}}^{\overline{x}} qdF(q) - \int_{\underline{x}}^{\overline{x}} qMC(q)dF(q) = 0. \qquad (3)$$

The equilibrium retail price is thus

$$p^S = \frac{1}{\int_{\underline{x}}^{\overline{x}} qdF(q)} \int_{\underline{x}}^{\overline{x}} qMC(q)dF(q) = \frac{1}{N} \int_{\underline{x}}^{\overline{x}} qMC(q)dF(q). \qquad (4)$$

The producer's expected short-run profit is

$$E(\pi^S) = \int_{\underline{x}}^{\overline{x}} qMC(q)dF(q) - \int_{\underline{x}}^{\overline{x}} C(q)dF(q). \qquad (5)$$

We can see that if the marginal production cost $MC(q)$ is increasing, then the producer could attain a higher price (for all its output) when demand is higher, and *vice versa*. This is a multiplicative effect on its revenue. Denote

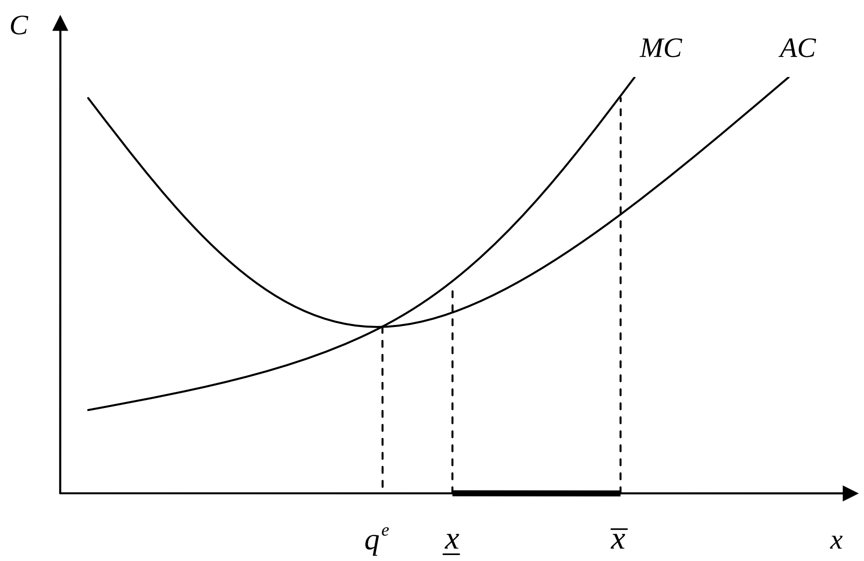
$$q^e \equiv \arg\min AC(q). \qquad (6)$$

When $q^e \in [\underline{x}, \overline{x}]$, we can write $E(\pi^S)$ as

$$E(\pi^S) = \int_{\underline{x}}^{q^e} q[MC(q) - AC(q)]dF(q) + \int_{q^e}^{\overline{x}} q[MC(q) - AC(q)]dF(q)]. \qquad (7)$$

If $MC(q)$ is increasing, then the first integral of (7) is negative because $MC \le AC$ on $[\underline{x}, q^e]$, and the second integral is positive because $MC \ge AC$ on $[q^e, \overline{x}]$. The sign of $E(\pi^S)$ depends on which term dominates. In particular, if the marginal production cost $MC(q)$ is strictly increasing, then the producer's profit under vertical separation is strictly negative (*i.e.* $E(\pi^S) < 0$) when $q^e > \overline{x}$, and $E(\pi^S) > 0$ when $q^e < \underline{x}$. Figure 1 illustrates the case when $q^e < \underline{x}$. Since the spot wholesale prices are always higher than the average production costs, the producer obtains positive profit. This is the case when the industry has "insufficient" production capacity.

Figure 1: Short-run equilibrium with insufficient capacity



2.3 Comparison

Now we compare expression (2) and (5) and see how the vertical structure affects the producer's profit in the short-run. Before presenting the main proposition of this section, we introduce a simple lemma about covariance, whose proof is omitted.

Lemma 1: Suppose *that* $f(.)$ *and* $g(.)$ *are two bounded continuous functions. For any random variable* x *on interval* $[\underline{x},\overline{x}]$, *we have*

$$Cov(f(x),g(x)) = E(f(x)\cdot g(x)) - E(f(x))\cdot E(g(x)) . \tag{8}$$

Proposition 1: *In the competitive electricity market, the producer's expected short-run profit under vertical integration,* π^I, *and that under vertical separation,* π^S, *satisfy*

$$E(\pi^S) = E(\pi^I) + Cov(q, MC(q)) . \tag{9}$$

Proof:

$$\begin{aligned} E(\pi^S) &= \int_{\underline{x}}^{\overline{x}} qMC(q)dF(q) - \int_{\underline{x}}^{\overline{x}} C(q)dF(q) \\ &= \int_{\underline{x}}^{\overline{x}} qMC(q)dF(q) - \int_{\underline{x}}^{\overline{x}} qdF(q)\cdot\int_{\underline{x}}^{\overline{x}} MC(q)dF(q) \\ &\qquad + \int_{\underline{x}}^{\overline{x}} qdF(q)\cdot\int_{\underline{x}}^{\overline{x}} MC(q)dF(q) - \int_{\underline{x}}^{\overline{x}} C(q)dF(q) \\ &= Cov(q, MC(q)) + E(\pi^I) . \end{aligned} \tag{12}$$

The last step uses Lemma 1 and expression (2). *Q.E.D.*

Proposition 1 suggests that if the covariance between electricity outputs and marginal production costs is positive, then the producer earns more profit under vertical separation than under vertical integration. Hence vertical separation is better for the producers in the short-run. The result is reversed if the covariance is negative. According to Schmidt (2003), a sufficient but not necessary condition for the covariance being positive is for the marginal production cost $C(q)$ being increasing. In the real world the marginal production cost for electricity is typically increasing, since high-cost generators are dispatched when demand is high. Hence vertical separation tends to increase power producers' profits in the short-run.

With increasing marginal production cost, vertical separation allows the producers to charge high prices when demand is high. This creates a multiplicative (high price multiplied by high quantity) effect on the producer's revenue. Although the producers receive low wholesale prices when demand is low, the impact on the profitability is relatively small because the quantity-supplied is low. Therefore the spot wholesale market actually serves as a commitment device for the producers to obtain more favorable terms of trading. The situation is reversed when the covariance is negative, since the spot wholesale price tends to be low when the demand turns out to be high.

The relationship between vertical separation and profitability reminds us of the "strategic vertical separation" in bilateral oligopoly markets, as addressed by Bonanno and Vickers (1988), Rey and Stiglitz (1988), and others. But the mechanisms are actually different. In a bilateral oligopoly with linear wholesale prices, vertical separation makes upstream firms perceive less elastic demands, and thus drives up the final prices. But in the current story, retailers have no market power, which means the demand elasticity argument does not apply.

From expressions (1) and (2), the equilibrium price under vertical integration is

$$p^I = \frac{1}{N}[\pi^I + \int_{\underline{x}}^{\bar{x}} C(q)dF(q)]. \tag{11}$$

And from expressions (4) and (5), the equilibrium retail price under vertical separation is

$$p^S = \frac{1}{N}[\pi^S + \int_{\underline{x}}^{\bar{x}} C(q)dF(q)]. \tag{12}$$

Hence $p^I > p^S$ if and only if $\pi^I > \pi^S$. Indeed, when demand is absolutely inelastic, the vertical structure only influences income distribution, but not social welfare in the short-run. Hence the consumers pay higher prices when and only when the producer makes more profits. Particularly, we have $\pi^I > \pi^S$ when $Cov(q, MC(q)) > 0$, which means the short-run electricity price is higher under vertical separation.

An example: Suppose that the aggregate demand may take two possible values, $x_1 = 50$ or $x_2 = 70$, each with probability of 0.5. The expected demand is thus 60. Suppose that the marginal production costs satisfy $MC(50) = 2$ and $MC(70) = 4$. The equilibrium price under vertical integration given by (1) is

$$p^* = 0.5MC(50) + 0.5MC(70) = 0.5 \times 2 + 0.5 \times 4 = 3 .$$

And the retail price under vertical separation given by (4) is

$$r^* = \frac{0.5 \cdot 50 \cdot MC(50) + 0.5 \cdot 70 \cdot MC(70)}{60} = \frac{0.5 \cdot 50 \cdot 2 + \cdot 0.5 \cdot 70 \cdot 4}{60} = \frac{19}{6} > 3 .$$

In this example, vertical separation raises the electricity price (or the profit of the producer) by 5.56% in the short-run.

2.4 Multiple competitive producers

We have assumed that there is one price-taking producer in the market. This might cause some confusion since it is certainly impossible for a single firm to be a price-taker. Nevertheless, the short-run analyses can be easily applied to the case with multiple homogenous producers when symmetric equilibria exist. Suppose there are m competitive producers. Each has a cost function of $C(q)$. The games played in the market are virtually the same as before.

In the symmetric equilibrium under vertical integration, the market price p^I (i.e., the marginal revenue from serving the last consumer) equals to the producers' expected marginal cost of serving the last consumer. Therefore we have $p^I = \int_{\underline{x}}^{\overline{x}} MC(\frac{q}{m}) dF(q)$. Each producer's expected sale is $\int_{\underline{x}}^{\overline{x}} \frac{q}{m} dF(q)$. Hence an individual producer's equilibrium profit can be written as

$$E(\pi^I) = \int_{\underline{x}}^{\overline{x}} \frac{q}{m} dF(q) \cdot \int_{\underline{x}}^{\overline{x}} MC(\frac{q}{m}) dF(q) - \int_{\underline{x}}^{\overline{x}} C(\frac{q}{m}) dF(q)$$
$$= \int_{\underline{x}/m}^{\overline{x}/m} q dF(mq) \cdot \int_{\underline{x}/m}^{\overline{x}/m} MC(q) dF(mq) - \int_{\underline{x}/m}^{\overline{x}/m} C(q) dF(mq) . \qquad (13)$$

Note that each producer faces demand distribution $F(mx)$ on interval $[\underline{x}', \overline{x}'] \equiv [\frac{\underline{x}}{m}, \frac{\overline{x}}{m}]$ in the symmetric equilibrium.

Under vertical separation, the spot wholesale price is $MC(\frac{q}{m})$ and each producer's output is $\frac{q}{m}$. Hence the equilibrium profit of a producer is

$$E(\pi^S) = \int_{\underline{x}}^{\overline{x}} \frac{q}{m} MC(\frac{q}{m}) dF(q) - \int_{\underline{x}}^{\overline{x}} C(\frac{q}{m}) dF(q)$$
$$= \int_{\underline{x}/m}^{\overline{x}/m} q MC(q) dF(mq) - \int_{\underline{x}/m}^{\overline{x}/m} C(q) dF(mq) . \qquad (14)$$

Comparing expression (13) and (14), one can see that the analyses are virtually parallel. In the case of symmetric short-run equilibrium, we do have the result that vertical separation leads to higher retail price and producer profits.

Finally, to prepare for the discussion of long-run equilibrium in the next section, we give a lemma about short-run equilibrium here.

Lemma 2: *In a symmetric short-run equilibrium under vertical integration, a producer's profit decreases with the number of producers in the market.*

Proof: First, the profit of a price-taking producer is non-decreasing with the market price. Second, under vertical integration the equilibrium price must decrease when a new producer enters the market, otherwise the aggregate quantity-supplied would exceed the aggregate quantity-demanded. Hence an entrant lowers the equilibrium profit of an incumbent producer. *Q.E.D.*

III. Long-run equilibrium

In the long-run electricity producers enter or exit the market until the equilibrium profits are driven to zero. Therefore the number of producers is endogenously determined. As in previous section we only consider symmetric equilibrium. Denote the equilibrium number of producers under vertical integration as m^I, and that under vertical separation as m^s.

3.1 Vertical integration

Under vertical integration, if a producer faces a demand that follows distribution $K(q)$ on interval $[\underline{y}, \overline{y}]$, then for given market price p the producer's profit is

$$E(\pi) = p\int_{\underline{y}}^{\overline{y}} qdK(q) - \int_{\underline{y}}^{\overline{y}} C(q)dK(q). \tag{15}$$

Define the "expected average cost" (McKean, 1969) of the producer as

$$\widetilde{AC} \equiv \frac{\int_{\underline{y}}^{\overline{y}} C(q)dK(q)}{\int_{\underline{y}}^{\overline{y}} qdK(q)}. \tag{16}$$

One can see from (15) and (16) that for given distribution function $K(q)$ and cost function $C(q)$, the price which allows zero profit for the producer is exactly $\widetilde{AC}$.

The long-run competitive equilibrium under vertical integration satisfies two conditions. First, each producer's expected average cost $\widetilde{AC}$ equals to the equilibrium price. Hence the producers can break even. Second, given cost function $C(q)$, each producer chooses the number of consumers such that its $\widetilde{AC}$ is minimized. Otherwise it would be replaced by other producers with lower $\widetilde{AC}$.

Denote the expected average cost of an integrated producer that serves *n* consumers as $\widetilde{AC}(n)$. Suppose that $\widetilde{AC}(.)$ is minimized at n^*, *i.e.*,

$$\widetilde{AC}(n^*) = \underset{n}{Min}\,\widetilde{AC}(n)\,. \tag{17}$$

In words, it is efficient for a producer to serve n^* consumers. As suggested above, the long-run equilibrium price must be

$$p^I = \widetilde{AC}(n^*)\,, \tag{18}$$

as long as the equilibrium exists. The equilibrium number of producers is thus

$$m^I = \frac{N}{n^*}\,. \tag{19}$$

Since each consumer's expected demand is 1, each producer's expected total cost is $n^*\widetilde{AC}(n^*)$, and the expected production cost of the industry is $N\widetilde{AC}(n^*)$. When the industry production is optimally allocated among the $\frac{N}{n^*}$ producers, the expected production cost of the industry is minimized. In other words, the long-run equilibrium under vertical integration is socially optimal.

3.2 Vertical Separation

Assume that each producer employs one price-taking retailer without loss of generality. The long-run equilibrium number of producers m^s allows the producers to break even. Hence m^s is determined by following zero-profit condition

$$E(\pi^S) = \int_{\underline{x}/m^S}^{\overline{x}/m^S} qMC(q)dF(m^S q) - \int_{\underline{x}/m^S}^{\overline{x}/m^S} C(q)dF(m^S q) = 0\,. \tag{20}$$

Given retail price p^S, a retailer also breaks even, *i.e.*,

$$p^S \int_{\underline{x}/m^S}^{\overline{x}/m^S} qdF(m^S q) - \int_{\underline{x}/m^S}^{\overline{x}/m^S} qMC(q)dF(m^S q) = 0\,. \tag{21}$$

From (20) and (21) we have

$$p^S \int_{\underline{x}/m^S}^{\overline{x}/m^S} qdF(m^S q) - \int_{\underline{x}/m^S}^{\overline{x}/m^S} C(q)dF(m^S q) = 0\,. \tag{22}$$

We can rewrite it as

$$p^S = \frac{\int_{\underline{x}/m^S}^{\overline{x}/m^S} C(q)dF(m^S q)}{\int_{\underline{x}/m^S}^{\overline{x}/m^S} qdF(m^S q)} = \widetilde{AC}(\frac{N}{m^S}) . \tag{22}$$

Hence the long-run equilibrium retail price under vertical separation equals to the expected average cost of each producer. Each producer serves $\frac{N}{m^S}$ consumers.

3.3 Comparison

Based on the discussion in previous subsections, we present the main result of this section in the following proposition.

Proposition 2: In *the long-run equilibrium under vertical separation, if* $Cov(q, MC(q)) > 0$, *there is excessive entry of producers. If* $Cov(q, MC(q)) < 0$, *there is insufficient entry of producers. In both cases, the equilibrium price is higher and the social surplus is lower under vertical separation.*

Proof: Consider first the case of $Cov(q, MC(q)) > 0$. Under vertical separation, each producer makes zero profit when there are m^s producers in the market. If there were m^s vertically integrated firms in the market, the firms would lose money in the short-run (Proposition 1). Therefore in the long-run, some firms would exit the market until a new equilibrium is achieved (Lemma 2). Hence we have $m^s > m^I$, which means vertical separation leads to excessive entry of producers.

The excessive entry implies that each producer serves less than n^* (defined by (17)) consumers. Hence the expected average cost under vertical separation, $\widetilde{AC}(\frac{N}{m^S})$, is higher than that under vertical integration. Hence the equilibrium retail price is higher and the social surplus is lower under vertical separation.

If $Cov(q, MC(q)) < 0$, it is similarly shown that there is insufficient entry of producers under vertical separation. Hence each producer serves more than n^* consumers and has expected average cost larger than $\widetilde{AC}(n^*)$. Hence vertical separation leads again to higher equilibrium retail price and lower social surplus. *Q.E.D.*

Since marginal production cost is typically increasing, vertical separation typically leads to excessive entry of producers or excessive investment in power generation in the real world. Proposition 2 shows that vertical integration in an electricity market improves the coordination between production and retail even when the market is perfectly competitive. As long as the transaction cost of integration is low enough, the vertical integration benefits consumers without hurting anyone else in the long run.

IV. Discussions

4.1 Fixed Retail Pricing

Consumers are supposed to only accept fixed prices in this paper. This assumption is usually satisfied in sales to small users, probably because fixed pricing is technically simple, and makes it easy for consumers to monitor their electricity expenditures. Actually the modeling of vertical separation is equivalent to that where producers directly sell to final consumers via variable prices that equal to the marginal production costs. If all consumers are willing to accept variable rates for their electricity consumption, the role of retailers becomes trivial. Hence from the perspective of producers, vertical separation does not make any difference.

Hence the current model can be viewed as considering the difference between fixed pricing, which is equivalent to the case of vertical integration, and variable pricing, which is equivalent to the case of vertical separation. It suggests that integrated electricity producers prefer variable pricing but consumers prefer fixed pricing in the short-run. The social surplus is higher under fixed pricing. In the long-run variable pricing induces excessive or insufficient entry of producers and therefore leads to higher average prices for consumers and lower social surplus.

Nevertheless, fixed pricing might be inefficient when the demand is elastic. For example, when the demand is high and thus the marginal production cost is high, it is socially desirable for consumers to accordingly reduce their usage. But fixed pricing cannot provide an economic incentive for them to do so. Therefore the net effect of fixing pricing on social welfare in the long-run is unsure.

4.2 Endogenous market structure

It can be shown that vertical separation cannot be an equilibrium outcome when there is no transaction cost in vertical integration. If producers can choose to sell directly to consumers or through retailers, from the equilibrium under vertical separation, an individual producer has incentive to integrate downward. Indeed, in the short-run equilibrium under vertical separation, each of the m producers serve $\frac{1}{m}$ of the consumers at price p^S. Suppose now one of the producers integrates downward. If the integrated producer maintains its market share, its profit is unchanged. However, an integrated producer is willing to serve $\frac{1}{m}$ of the consumers only when the retail price equals to the short-run equilibrium price under vertical integration p^I, which is less than p^S when $Cov(q, MC(q)) > 0$. Hence at

p^S the integrated producer wishes to sign up more consumers and makes more profits. Similarly, if $Cov(q, MC(q)) < 0$, which means $p^S < p^I$, the integrated producer wishes to sign up less consumers and makes more profits. Hence deviating to vertical integration is always profitable in the absence of transaction costs, which means the vertical separation cannot be an equilibrium outcome.

On the contrary, suppose the *status quo* is the equilibrium outcome under vertical integration, with retail price p^I. In order to observe if an integrated electricity firm has incentive to go vertically separated, we need to assume that there is a competitive wholesale market where the spot wholesale prices equal to marginal production costs. When $Cov(q, MC(q)) > 0$, an independent retailer cannot break even with retailer price p^I, which is less than p^S. Hence the separation is not feasible. When $Cov(q, MC(q)) < 0$, an independent retailer makes positive profit with retail price p^I, which is higher than p^S. Since the total profit of the producer and retailer is unchanged, the producer necessarily loses from selling through the wholesale market. Therefore an integrated electricity firm does not have incentive to go vertically separated.

To sum up, vertical integration is the only possible vertical market structure in equilibrium when there is no transaction cost in vertical integration. In particular, it is impossible to have a "hybrid" market structure where producers can choose to sell through a spot wholesale market or directly to final consumers. This finding might not be consistent with what we observe in the real world. Indeed, spot wholesale market may exist for various reasons, *e.g.*, electricity firms are heterogenous, or vertical integration is costly. Rigorous modeling of those cases is left for future studies.

As suggested before, this paper can also be viewed as comparing the difference between fixed pricing and variable pricing. If there were no transaction costs, it can be similarly shown that the pricing form of the producers is endogenous. With absolutely inelastic demand, market competition would drive the producers to choose fixed pricing rather than variable pricing.

4.3 Demand elasticity

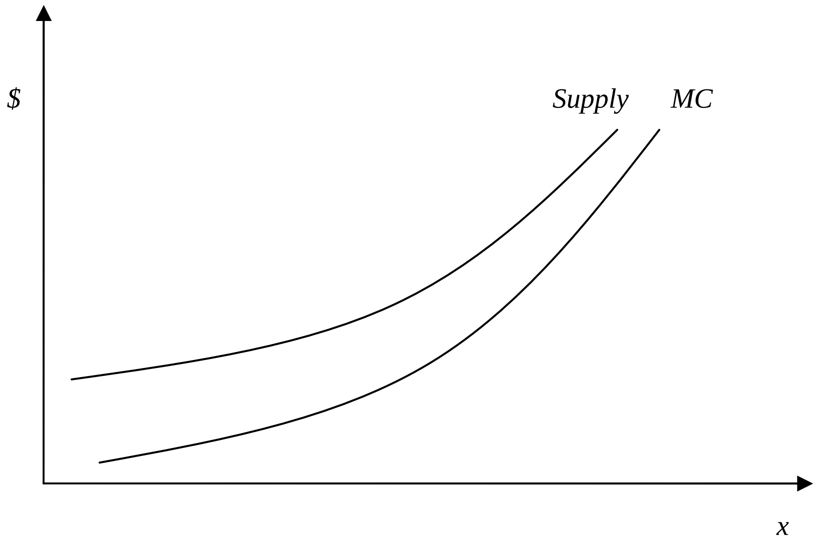
We have assumed that consumer demands are absolutely inelastic. This assumption is not crucial as long as the retail price is fixed. If the demand curve is downward-sloping, then for a given retail price p the quantity demanded $x(p)$ follows a cumulative distribution $F(x;p)$ on an interval $[\underline{x},\overline{x}]\subseteq R^{+}$. The decreasing demand function implies that $\frac{\partial F(x;p)}{\partial p}>0$ for any $x\in[\underline{x},\overline{x}]$. Given the number of producers in the market, still denote the short-run equilibrium price as p^I (under vertical integration) or p^S (under vertical separation). Suppose that $Cov(q,MC(q))>0$. Under vertical integration, the market is cleared at price p^I. At price p^I, the (uncertain) market demand is $x=x(p^I)$. Therefore it must be the case with demand $x=x(p^I)$, the short-run equilibrium price under vertical integration is exactly p^I, otherwise the market cannot be cleared. According to Proposition 1, with demand $x=x(p^I)$, the short-run equilibrium price under vertical separation must be higher than p^I, because price p^I is not high enough to clear the market under vertical separation. Hence with the original elastic demand $x=x(p)$, the equilibrium price p^S must be higher than p^I. Vertical separation generally leads to more profits to the producers in the short-run. The story is reversed if the covariance is negative. Hence we can obtain similar results about vertical separation when the demand curve is downward-sloping.

4.4 Market power

This paper only considers the case of perfect competition. Indeed, competition is crucial for the success of an electricity industry reform because market power in this market often invites significant price increase and/or opportunistic behaviors. Nevertheless, real-world electricity firms often have some market power. For instance, in the UK in 1993, the market shares of three investor-owned generators, National Power, Powergen, and Nuclear Electric, were 52%, 33% and 15% respectively. Such a high level of market concentration might have helped to generate the disappointing performance of the UK deregulation before 2001. In contrast, the largest power producer of the Nordic market, Vattenfall, only takes a market share of 17%. Vattenfall is wholly owned by Swedish government.

In a market with oligopolistic vertically-integrated producers, the producers may sell to consumers via fixed prices or variable prices. In the case of variable pricing, the producers typically charge high prices in high demand states, and *vice versa*. In practice producers typically compete by choosing supply functions (hour-ahead, day-ahead, *etc*.). The cases fall to the models of Klemperer and Meyer (1989) or von der Fehr and Harbord (1993). When producers have market power, their supply curves should be above their marginal cost curves, as shown in Figure 2. The smaller the market powers are, the closer the supply curves are to the marginal cost curves.

Figure 2: Supply curve of an oligopolistic producer



Based on the model of Klemperer and Meyer (1989), one might consider the case where oligopolistic producers offer supply curves to competitive retailers. The retailers sell to consumers via fixed prices. In such a model the producers have to take the zero profit condition of the retailers into consideration. We conjecture that vertical integration not only internalizes the externalities caused by the strategic interaction between producers and retailers (Winter, 1993; and others), but also improves the vertical coordination of the industry. Details are left for future studies.

V. Concluding Remarks

This paper captures two critical features of electricity markets, which are the non-storability of the goods and the uncertainty of the aggregate demand. The main argument is that the separation of production and retail does affect the performance of a competitive electricity market. The model demonstrates that when the covariance between electricity outputs and marginal production costs is positive, then the separation of production and retail results in higher electricity price and producer profits in the short-run. The reverse is true when the covariance is negative. In the long-run, vertical separation induces excessive or insufficient entry of power producers to the market, and thus results in higher electricity price and lower social surplus. The findings suggest that governments should encourage vertical integration or long-term contractual relationship between electricity producers and retailers, when the associated transaction costs are not too high.

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